CHAPTER -17

MEADE'S MODEL OF GROWTH

Like the models of Solow and Swan, Meade's model is also neo-classical in nature where the production function is such that different factors of production are substitutable in production. The warranted rate of growth is a variable and it adjusts with the natural rate of growth to maintain steady state equilibrium.

17.1. Assumptions of the Model

Meade's model is based on the following assumptions : (1) The economy is a closed economy with no activities on the part of the government. (2) There is perfect competition in commodity and factor markets. (3) The production function is homogeneous of degree one which means that there are constant returns to scale in the production process. (4) In the economy only one commodity is produced which may be used either for final consumption or for additions to the stock of capital. This assumption is called the assumption of perfect substitutability in production between capital goods and consumption goods. (5) There are three factors of production, land, labour and the stock of machines. Supply of land is assumed to be fixed. (6) Production functions are such that one factor of production can be substituted for another. (7) All capital goods (*i.e.,* machines) are alike (they are simply a ton of steel) and that the ratio of labour to machinery can be varied with equal ease in the short run as in the long run. This assumption is called the assumption of perfect malleability of machinery. (8) Full employment of land, labour and capital is achieved by the adjustment of wage-price flexibility. (9) It is assumed that there is no depreciation of capital. (10) Technical progress takes place at a constant rate.

17.2. Three Determinants of the Rate of Economic Growth

The aggregate production function of the economy can be written as $Y = F(K, L, N, t)$ where Y = net output or net national income, K = the existing stock of machines, L = the amount of labour, N = the amount of land, t = time. Since the mere passage of time brings technical progress and allows Y to be raised even without any increase in K, L or N, t may be regarded as an index of technical progress.

Let V be the marginal physical product of capital. It is the additional output that can be obtained by employing one additional unit of capital stock, other factors of production remaining the same. Similarly, let W be the marginal physical product of labour. We assume that total land is constant. Hence, output can increase either due to change in capital stock, or due to change in labour force or due to technical progress. $\Delta Y = V.\Delta K + W.\Delta L + \Delta Y'$ where $V.\Delta K$ in the increase in output caused by the increase in capital stock, $W.\Delta L$ is the increase in output due to increase in labour force and $\Delta Y'$ is the increment in output due to technical progress. This basic relationship can be written as

$$\frac{\Delta Y}{Y} = \frac{VK}{Y}.\frac{\Delta K}{K} + \frac{WL}{Y}.\frac{\Delta L}{L} + \frac{\Delta Y'}{Y}$$

where $\frac{\Delta Y}{Y}$ = rate of growth of output, $\frac{\Delta K}{K}$ = rate of growth of capital, $\frac{\Delta L}{L}$ = rate of growth of labour, $\frac{\Delta Y'}{Y}$ = rate of technical progress.

Let us denote these four rates by y, k, l and r respectively.

$\frac{VK}{Y} = \frac{\Delta Y}{\Delta K}.\frac{K}{Y}$ is the elasticity of output with respect to capital stock. It is also equal to the proportion of national income going to the owners of capital if they receive a reward equal to the marginal product of capital. Similarly, $\frac{WL}{Y}$ is the output elasticity with respect to labour or the proportion of national income going to labourers if they receive a wage rate equal to the marginal productivity of labour. Let us denote $\frac{VK}{Y}$ by U and $\frac{WL}{Y}$ by Q. They are called the proportional marginal products of capital and labour respectively.

The basic relationship can therefore be written as

$y = Uk + Ql + r$. This shows that the growth rate of output (y) is the weighted sum of the three other growth rates, namely the growth rate in the stock of capital (k), the growth rate of population (l) and the growth rate of technical progress (r). The growth rate of capital is weighted by the proportional marginal product of capital (U) which shows the marginal importance of capital in the production process. Similarly, the growth rate of labour is weighted by the proportional marginal product of labour which shows the marginal importance of labour in the production process.

We can write the basic relationship $y = Uk + Ql + r$ in the following form : $y - l = Uk - (1-Q)l + r$. Now, $y - l$ is the difference between the growth rate of total output and the growth rate of total population and it measures the growth rate of per capita output. The above relationship shows that the rate of growth of per capita income is the outcome of three factors ; *first,* it is raised by the growth rate of real capital (k) weighted by its proportional marginal product ; *second,* it is depressed by the growth rate of the working population weighted by $(1 - Q)$ and *third,* it is raised by the rate of technical progress. The second of these three elements is the familiar tendency for diminishing returns to labour to set in as more and more labour is applied to any given amount of land and capital.

But the diminishing returns to labour may be offset by increasing returns to scale. Let us consider this possibility in some details. Consider the production function $Y = F(K, L, N, t)$ and assume that t is a constant *i.e.,* we are considering any point of time. Here, $\Delta Y = V.\Delta K + W.\Delta L + G.\Delta N$ where G is the marginal product of land and ΔN is the additional land. Let Z represent the elasticity of Y with respect to land. Then we know from Euler's theorem on homogeneous production function that under conditions of constant returns to scale $U + Q + Z = 1$ and in the case of increasing returns to scale $U + Q + Z > 1$ whereas in the case of decreasing returns to scale $U + Q + Z < 1$.

Let us now consider our basic relationship $y - l = Uk - (l - Q)l + r$. If there are constant returns to scale and $U+Q+Z=1$, then Q is certainly less than unity, the term $-(1-Q)l$ is certainly negative and the growth rate in real income is certainly depressed by the growth rate of population. Similar will also be the case if there are decreasing returns to scale. But if there are increasing returns to scale, $U + Q + Z > 1$ and all we know is that $Q > 1 - U - Z$. It is now possible that $Q > 1$. This is more likely to be so, first, the more important are the increasing returns to scale (*i.e.,* the more Q exceeds $1-U-Z$) and second, the smaller are the proportional marginal productivities of capital and land (*i.e.,* the smaller are U and Z).

Let us consider the factors that will make Z low. Z can be low in two cases : In the *first* place land may be very plentiful but the substitutability between land and other factors in production may be so small that its marginal product is very low. In this case land is a free good. Here land is important but its marginal product is so low that its proportional marginal product is very low. In the *second* place, suppose that very little land is available so that the marginal product of land is high. But there is a fair degree of substitutability between land and other factors so that the importance of land is very low. In this case also Z will be very low.

In the simple case where land is a free good and where there are constant returns to scale $Q+U=1$. In this case $y-l=U(k-l)+r$ so that the growth rate of real income per head would equal the growth rate in the amount of capital per head $(k-l)$ weighted by the proportional marginal product of capital plus the rate of technical progress.

In our basic relationship the element Uk can be expressed in other forms. $U=\frac{VK}{Y}$ and $k=\frac{\Delta K}{K}=\frac{I}{K}=\frac{SY}{K}$ where I = investment $=\Delta K$, S = proportion of national income saved. Again, if we put $\frac{\Delta Y}{\Delta K}=V$ then $US\frac{Y}{K}=\frac{\Delta Y}{\Delta K}.\frac{K}{Y}.S.\frac{Y}{K}=S.\frac{\Delta Y}{\Delta K}=SV$. Then the basic relationship can be written in three different ways :

(1) $y-l=Uk-(1-Q)l+r$

(2) $y-l=US\frac{Y}{K}-(1-Q)l+r$

(3) $y-l=SV-(1-Q)l+r$

These are all three ways of saying identically the same thing.

17.3. Changes in the Rate of Economic Growth

We have considered the main factors which determine the growth rate of real income per capita. Let us now see in what conditions the growth rate of real income per capita is itself rising or falling over time. Consider the basic growth relationship in the form

$y-l=SV-(1-Q)l+r$. Let us assume that l and r are determined by non-economic exogenous factors and that both these growth rates are themselves constant over time.

With l and r given and constant, it is clear from our basic relationship that whether or not $y-l$ will be rising or falling over time can be discussed in terms of what is happening to the values of V, S and Q over time. There are four major sets of considerations which will determine what is happening to V, S and Q over time.

First, in the absence of technical progress and of population growth a given rate of capital accumulation will be raising the amount of capital per head and, because of diminishing returns to a single factor of production, the marginal rate of return on real capital (V) will be falling. The element SV in our basic relationship will therefore be falling. Moreover, a high S will cause a rapid rate of fall in V, since the more rapidly real capital accumulates the more the marginal return on it falls. The fall in V will be slower, the greater the elasticity of substitution between capital and other factors of production.

Second, the rate of technical progress will tend to offset this effect upon the level of V. A rapid rate of technical progress (r) will tend to raise V over time.

Third, the nature as well as the amount of technical progress will affect the growth rate of real income per capita over time. A rapid rate of technical progress may be expected to raise the marginal products of all factors of production simultaneously. But technical progress may be biased in making one factor more or less important at the margin as time passes. Suppose that in an economy capital is being accumulated more quickly than population is growing. If, at the same time, technical progress is biased in the direction of making capital more important relative to labour at the margin of production as time passes, then this will be a further factor tending to raise the rate of growth of output per capita over time.

Fourth, the proportion of national income which is saved (S) may itself rise or fall because of a change in the distribution of income. Let us take an example. Suppose that (*i*) a larger proportion of profits than of wages is saved, (*ii*) that capital is being accumulated at a higher rate than population is growing, (*iii*) that there is a high elasticity of substitution between capital and labour, and (*iv*) that technical progress is of the type which makes capital relatively more important and labour relatively less important at the margin of production. Then, in these conditions, as time passes, a larger and larger proportion of national income is likely to go to profits and a smaller and smaller proportion to wages. As a result, the proportion of the national income which is saved (S) will tend to rise over time. Hence, the element VS in the determination of the growth rate in the per capita output will tend to rise over time.

17.4. Definition of the Nature and the Rate of Technical Progress

Suppose that in the course of a year technical knowledge so improves that, with an unchanged amount of land, labour and capital, 2 percent more output can be produced. Then we say that the rate of technical progress is 2% per annum. But suppose that the marginal product of labour is raised by 3% when there is a technical progress at the rate of 2%. In this case technical progress is said to be labour-using in character. Conversely, if the technical progress raises the marginal product of labour by less than the rate of technical progress, then it is said to be labour-saving in character. Similar is the definition of technical progress being land-using or land-saving and capital-using or capital-saving.

Alternatively we can define technical progress as labour-using or labour-saving according as it tends to raise or lower the proportional marginal product of labour, Q. By definition $Q = \frac{W.L}{Y}$ where W is the marginal product of labour. If with a constant amount of all the factors both the total product (Y) and the marginal product of labour (W) go up in the same proportion, then the proportional marginal product of labour (Q) will remain constant. If technical progress raises W at a greater proportion than Y, technical progress will be labour-using and Q will rise. Similarly, labour-saving technical progress will tend to reduce Q, capital-using technical progress will tend to raise U, capital-saving technical progress will lower U, land-using technical progress will raise Z and land-saving technical progress will lower Z.

Under conditions of constant returns to scale $U + Q + Z = 1$. It follows therefore that technical progress may be altogether neutral *i.e.*, it may neither 'save' nor 'use' any factor, so that it leaves U, Q and Z unchanged. But if it is biased in the direction of 'saving' one factor, it must be simultaneously biased in the direction of using another factor and *vice versa*. For example, if it tends to lower Q it must tend to raise U or S in an offsetting manner. Even if there are increasing returns to scale in the sense that $U+Q+Z$ is greater than unity, it is still true provided that the degree of increasing returns to scale is constant. But if the technical progress is of a kind to increase or decrease the degree of returns to scale itself, then it may be biased in the direction of using or saving all factors at the same time.

The nature of technical progress can also be understood with the help of a diagram. Suppose that population is constant; then $l = 0$ and $y = VS + r$. If r is constant, the problem is whether VS will be increasing or not as a result of technical progress. This problem is illustrated with the help of the following diagram (figure - 17.1). We measure total output (Y) on the vertical axis and total capital stock (K) on the horizontal axis. We assume that other factors of production are constant. At any one time we can draw a curve such as $F_1(K)$ which shows the amount of output (Y) that can be achieved by any given capital stock (K). For example, if in year 1, capital stock is OD, output is AD. Slope of the curve $F_1(K)$ at A is the marginal product of capital (V). We assume that as more of capital is employed, the marginal product of capital decreases. That is, marginal product of capital at G is less than that at A and the $F_1(K)$ curve is concave from below.

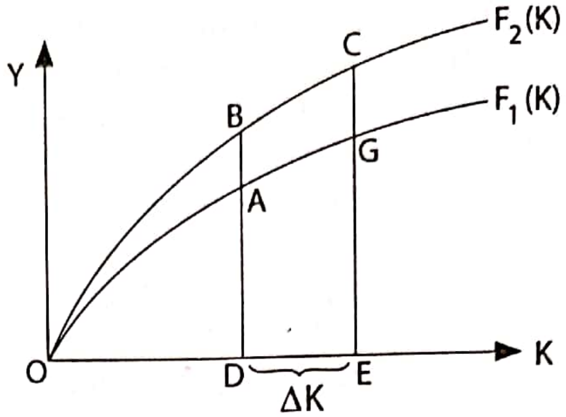

Fig 17.1

Between year 1 and year 2 technical progress will have taken place. In year 2 the total product curve shifts to $F_2(K)$. It represents that by employing the same amount of capital (OD) more output (BD) can be produced. $\dfrac{AB}{AD}$ is the rate of technical progress by definition. This technical progress will be capital-using (or capital-saving) if it raises the marginal product of capital in a proportion which is greater (or, less) than the rate of technical progress itself. In other words, if technical progress is neutral, the slope of F_2 at B will be greater than the slope of F_1 at A in the same proportion as the output BD is greater than the output AD. Between A and B the rate of profit (V) will have risen in the ratio $\dfrac{AB}{AD}$ if technical progress is neutral.

But between year 1 and year 2 capital will have been accumulated. Let us suppose that the capital stock has increased to OE. Then in year 2 we shall in fact move from A to C. Because of diminishing returns to a single factor the slope of F_2 curve at C will be lower than that at B. Hence to know whether V is rising or falling through time, we need to know whether the slope of F_2 at C is greater or less than the slope of F_1 at A. The slope of F_2 at C is likely to be greater than the slope of F_1 at A; (*i*) the greater is the rate of technical progress, (*ii*) the more capital-using is the nature of technical progress, (*iii*) the lower is the proportion of income saved and (*iv*) the more readily capital can be substituted for land and labour in production. Whether V will be rising or falling through time is the net result of the four factors listed above.

In summary we can say that the following factors will lead to an increase in the rate of growth of income (assuming that population is constant) : (*i*) a high rate of technical progress,

(*ii*) a capital-using type of technical progress,

(*iii*) a low initial proportion of income saved,

(*iv*) a high degree of substitutability between capital on the one hand and land and labour on the other hand,

(*v*) a rising proportion of income saved as real income rises, and

(*vi*) an especially high proportion of profits devoted to saving.

17.5. The State of Steady Economic Growth

Let us now assume that population is growing at a constant rate, l. We also assume that the rate of technical progress (r) is also a constant. With a constant l, the rate of growth of per capita income $(y-l)$ will be constant only if the growth rate in total output (y) is constant. We shall now prove that *the growth rate of total output will always move towards a constant level if* : (*i*) all elasticities of substitution between the various factors are equal to unity, (*ii*) technical progress is neutral towards all factors, and (*iii*) the proportions of profits saved, of wages saved and of rents saved were all three constants.

This can be proved as follows : conditions (*i*) and (*ii*) would mean that the proportions of national income going to profits, wages and rents (namely U, Q and Z) would remain constant during the process of growth. Now let S_v represent the proportion of profits saved. Total profit = UY and therefore total saving out of profits = S_vUY. Similarly, S_wQY is total saving out of wages and S_gZY is total saving out of rents (where S_w = proportion of wages saved and S_g = proportion of rents saved). But total saving is equal to SY. Therefore by definition,

$$SY = S_vUY + S_wQY + S_gZY$$

$$\therefore\ S = S_vU + S_wQ + S_gZ$$

By conditions (*i*) and (*ii*), U, Q and Z are constants. Further, by condition (*iii*) S_v, S_w and S_g are also constants. Therefore S is also constant.

Now, consider our basic relationship $y = Uk + Ql + r$. Note that U, Q, l and r are constants. Hence y will be constant only if k is constant. But k = rate of growth of capital stock = $\frac{SY}{K}$. But since S is constant, k will be constant only if Y and K both grow at the same rate per annum or in other words $\frac{Y}{K}$ will be constant if $y=k$.

We have thus reached the conclusion that *if the growth rate of capital stock is equal to the growth rate of national income, then the growth rate of national income will be constant.*

In order, therefore, to understand the conditions in which the growth rate of income will be constant we have to investigate simply the conditions in which the growth rate of capital stock will be equal to the growth rate of national income. Consider the basic relationship $y = Uk + Ql + r$

Let $y = k = a$. Then

$a = Ua + Ql + r$

or, $a(1-U) = Ql + r$

$\therefore\ a = \dfrac{Ql+r}{1-U}.$

In other words, if the growth rate of capital stock is $\frac{Ql+r}{1-U}$, then the growth rate of national income will also be equal to $\frac{Ql+r}{1-U}$.

Now the growth rate of the capital stock will initially be at this critical level of $\frac{Ql+r}{1-U}$ only by a mere fluke of chance. What will happen if the rate of accumulation of capital stock is greater than or less than this critical rate ? Suppose that

$k > \frac{Ql+r}{1-U}$ *i.e.,* $S\frac{Y}{K} > \frac{Ql+r}{1-U}$. Since $\frac{Ql+r}{1-U}$ is the rate of accumulation of capital which will cause income (Y) and capital stock (K) to grow at the same proportionate rate, rate of accumulation of capital greater than this will mean that capital stock will grow at a greater proportionate rate than income. Therefore, so long $\frac{SY}{K} > \frac{Ql+r}{1-U}$, Y will be growing at a lower proportionate rate than K, and $\frac{Y}{K}$ will be falling. Since S is constant $S.\frac{Y}{K}$ will also be falling. This process will continue so long $S.\frac{Y}{K}$ remains greater than $\frac{Ql+r}{1-U}$. Similarly, it can be seen that if k started a figure below the critical rate $\frac{Ql+r}{1-U}$, then income will grow more quickly than capital so that $\frac{Y}{K}$ will rise and $k\left(=S\frac{Y}{K}\right)$ will rise towards the level $\frac{Ql+r}{1-U}$.

We may, therefore, conclude that if the growth rate of population is constant, if the three elasticities of substitution between the three factors land, labour and capital are all unity, if technical progress is neutral and is at a constant rate and if the proportions of profits, wages and rents saved are constant, then the growth rate of real income and the capital stock will both tend towards a constant level equal to $\frac{Ql+r}{1-U}$.

The basic argument can be expressed by means of a simple diagram (figure 17.2).

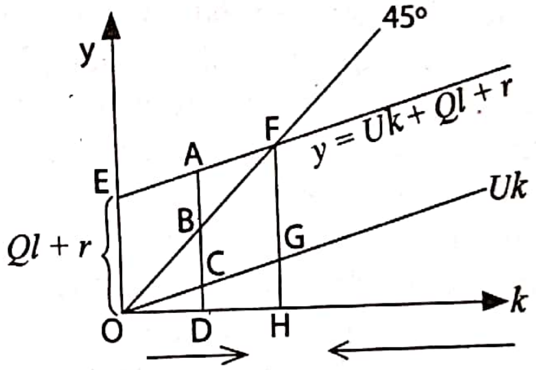

Fig 17.2

On the horizontal axis we plot k while on the vertical axis we plot y. Through the origin draw the line OCG with a slope equal to U. Next measure up the vertical axis a length OE equal to $Ql+r$. Draw through E the line EAF parallel to OCG. The height of the line EAF then represents $y=Uk+Ql+r$. Thus when $k=OD$, $Uk=CD$ and $Ql+r=AC(=OE)$ so that $AD=CD+AC=Uk+Ql+r$. Finally draw a 45° line through O. Consider the point of intersection, F, of the 45° line with the EAF line. At point F, $OH = FH$ so that $k=Uk+Ql+r$ *i.e.*, $k(1-U)=Ql+r$ and $k=\frac{Ql+r}{1-U}$. Thus point F represents the state of steady growth at which $y=k=\frac{Ql+r}{1-U}$. Now suppose that the value of k is less than OH. Suppose that $k = OD = BD$. Here $y = AD$. Since $AD > BD$ this means that $y > k$. This implies that $\frac{Y}{K}$ will be rising. Hence, $k=\frac{SY}{K}$ will also be rising towards OH. Similar argument stands if we start with a value of $k > OH$.

Consider a special case when there is no technical progress, there are two factors of production labour and capital and the production function is subject to constant returns to scale. In this case $r = 0$ and $Q + U = 1$ so that $Q = 1 - U$. In this case the condition of steady growth reduces to $y = k = l$ which represents the familiar equality between the warranted and natural rates of growth found in the neo-classical growth models of Solow and Swan.

17.6. An Alternative Treatment of Technical Progress

So long technical progress is defined as being neutral if, with unchanged supplies of all the factors, the marginal product of every factor is raised in the same proportion. This definition stems from that used by Prof. Hicks in his *Theory of Wages.* An alternative definition has also been very widely used by writers on the theory of economic growth according to which technical progress is neutral if the rate of profit remains constant when the ratio of capital stock to national income remains constant. This definition stems from that used by Harrod in his *Towards a Dynamic Economics.*

According to Prof. Meade the Hicksian definition is better than the Harrodian definition on two grounds. *First,* it provides a water tight definition even when there are a large number of factors of production. Harrodian definition, however, is imprecise when there are more than two factors of production. *Second,* the use of the Harrodian definition of neutral technical progress almost inevitably involves a measurement of the rate of technical progress in terms of the growth rate of output which would occur not with a constant stock of capital but either (*i*) with the capital stock growing at the same rate as output (so that the capital-output ratio is constant), or (*ii*) with the capital stock growing at a rate just sufficient to keep the rate of profit constant.

We shall now explain the difference between the two types of definitions and their implications. For this let us assume that there are only two factors of production–labour and capital, and that population is in fact constant. We are then considering economic growth which is due either to the accumulation of

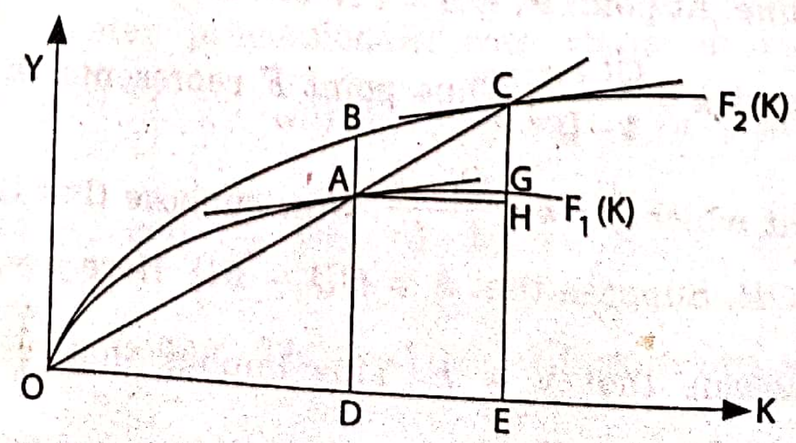

Fig 17.3

CHAPTER -18

JOAN ROBINSON'S MODEL

18.1. Introduction

Mrs. Joan Robinson's model is a neo-classical model which derives the condition of steady state equilibrium ('golden age' in her terminology). Mrs. Robinson's ideas are expressed in verbal terms in two books : *The Accumulation of Capital* and *Essays in the Theory of Economic Growth.* She did not build any formal model in terms of equations. The model considered here is due to Kurihara in his book, *The Keynesian Theory of Economic Development.* The essence of Mrs. Robinson's theory can be summed up in her central proposition : "If they have no profit, the entrepreneurs cannot accumulate, and if they do not accumulate they have no profit." She is interested in explaining the fundamental nature of economic growth according to the capitalist rules of the game.

18.2. Assumptions of the Model

The model is based on the following assumptions :

1. The economy is a closed economy with no economic activities on the part of the government.
2. The economy produces only one commodity which can act both as a means of consumption and as a means of production.
3. Only two factors of production – labour and capital – are used in the production process.
4. The whole product is distributed between the entrepreneurs and the wage earners.
5. Wage earners spend all of their wage income on consumption while profit earners save and invest all of their profit income. This means that workers save nothing while entrepreneurs consume nothing.
6. In the production process capital and labour are used in a fixed proportion. This assumption is later relaxed to allow for the complications of reality.
7. There is no technical progress.

18.3. The Structure of the Model

The basic equation giving the distribution of total output between workers and entrepreneurs is given by

$pY = wN + \pi \, pK$ ..... (1)

where Y is national output, N the amount of labour employed, K

the amount of capital utilised, p the average price of output as well as of capital equipment, w the money wage rate and π the gross profit rate. Dividing both sides of (1) by p we get the distribution equation in real terms :

$$Y = \frac{w}{p} N + \pi K \quad ..... (2)$$

From this equation we can determine the profit rate (π) as follows :

$$\pi = \frac{Y - \frac{w}{p} N}{K} = \frac{\frac{Y}{N} - \frac{w}{p}}{\frac{K}{N}}, \text{ or, } \pi = \frac{\rho - \frac{w}{p}}{\theta} \quad ..... (3)$$

where $\rho = \frac{Y}{N}$ represents the average productivity of labour and $\theta = \frac{K}{N}$ represents the capital - labour ratio.

Equation (3) shows that the profit rate depends on three factors : average productivity of labour, the real wage rate and the capital - labour ratio. The expression $\rho - \frac{w}{p}$ represents net return from one unit of labour because ρ is the output obtained fron one unit of labour while $\frac{w}{p}$ is the real wage paid to one unit of labour. Equation (3) shows that the rate of profit varies directly with the net return from one unit of labour and varies inversely with the capital-labour ratio.

The production function is given by $Y = F(K, N)$ ..... (4)

This function is assumed to be homogeneous of degree one *i.e.* subject to constant returns to scale. Equation (4) is the production counterpart of distribution equation (1).

Consider now the expenditure side. In equilibrium national output will be equal to sum of real consumption expenditure (C) and real investment expenditure (I).

$Y = C + I$ ..... (5)

or, $Y - C = I$ or, $S = I$ which is the Keynesian saving investment equation. Since it has been assumed that workers consume the whole of their wage and entrepreneurs do not consume anything, it follows that

$C = \frac{\omega}{p}.N$ ..... (6)

i.e. Consumption expenditure in real terms is equal to total real wage bill. Again total real saving is equal to total profit income in real terms

i.e. $S = \pi K$ ..... (7)

Since investment is nothing but addition to the stock of capital

$I = \Delta K$ ..... (8)

From (7) and (8) we get

$\Delta K = \pi K$..... (9) [$\because I = S$]

$$\therefore \frac{\Delta K}{K} = \pi = \frac{\rho - \frac{w}{p}}{\theta} \text{ ..... (10)}$$

The rate of growth of capital given by (10) is the rate which is obtainable by the entrepreneurs following the capitalist rules of the game. Equation (10) shows that the rate of growth of capital can be increased if the net return to one unit of labour $\left(\rho - \frac{w}{p}\right)$ rises in greater proportion than the capital-labour ratio. If the technological conditions of production as embodied in ρ and θ remain the same, then the rate of growth of capital increases when the real wage rate falls and decreases when the real wage rate rises. According to Kurihara, "J. Robinson has brought us back to David Ricardo's theory of economic development, albeit via the Keynesian door." It is also seen from (10) that the rate of growth of capital is determined by the same factors which determine the rate of profit.

18.4. The Golden Age

Mrs. Robinson defines 'Golden age' as a situation of equilibrium with full employment of labour and full utilisation of capital. If the capital - labour ratio, $K/N = \theta$, is constant in conditions of full employment and full utilisation, an increase in the amount of fully employed labour is given by $\Delta N = \Delta K/\theta$. The rate of growth of fully employed labour is then given by

$$\frac{\Delta N}{N} = \frac{\Delta K/\theta}{N} = \frac{\Delta K/\theta}{K/\theta} = \frac{\Delta K}{K} \text{ ..... (11)}$$

This means that fully employed labour grows at the same rate as the rate of growth of capital, when the capital-labour ratio (θ) remains constant. Equation (11) signifies a golden age equilibrium with full employment of both labour and capital.

Mrs. Robinson considers the question as to whether the economy possesses any equilibrating mechanism if and when it diverges from the golden age equilibrium for some reason. Suppose $\frac{\Delta N}{N} > \frac{\Delta K}{K}$ *i.e.* Labour force is growing faster than capital stock. Whether or not the economy can get back on the path of golden age equilibrium depends, in Robinson's view, on the behaviour of the profit-wage relation. Given the state of technology, a higher rate of growth of labour compared to the rate of growth of capital leads to a reduction of the money wage rate. If the general price level remains constant, the real wage rate (w/p) will fall. If this happens the rate of growth of capital increases. The amount by which the rate of growth of capital increases due to fall in the real wage rate is found from (10). If, however, the real wage rate fails to fall either because money wages are rigid or because the price level falls in the same proportion as money wages, the equilibrating mechanism cannot work and progressive under employment cannot disappear. This possibility is similar to Harrod's divergence between warranted rate and natural rate which results from fixed factor proportion.

Consider now the case when $\frac{\Delta K}{K} > \frac{\Delta N}{N}$ *i.e.* when the rate of growth of capital is greater than the rate of growth of labour. In this case there will be scarcity of labour and the real wage rate will increase. As the real wage rate increases, the rate of profit and the rate of growth of capital fall. This will continue until $\frac{\Delta K}{K}$ becomes equal to $\frac{\Delta N}{N}$. Even though the real wage rate is rigid, a change in labour productivity (ρ) or in the capital-labour ratio (θ) might be such as to decrease the profit rate and hence the rate of growth of capital. A change in labour productivity or a change in the capital labour ratio is equivalent to a shift of the production function.

18.5. Relationship of Robinson's Model to Harrod-Domar Model

The relation of the Robinson model to those of Harrod and Domar can be seen as follows : From (3) we get

$$\pi = \frac{Y - \frac{\omega}{p} N}{K} = \frac{Y}{K}\left(\frac{Y - \frac{\omega}{p} N}{Y}\right).$$

Now $\frac{Y}{K}$ is the average productivity of capital. If there is fixed proportionality between output and capital then it is also the marginal productivity of capital. On the other hand $\frac{Y - \frac{\omega}{p}N}{Y}$ represents the share of profits in national income. Since $S = \pi K$ and $\frac{Y - (\omega / p).N}{Y} = \frac{\pi K}{Y}$, therefore $\frac{Y - \left(\frac{\omega}{p}\right)N}{Y} = \frac{S}{Y} = s$. Then $\pi = \frac{\Delta K}{K} = \frac{Y}{K}.s$.

If $\frac{Y}{K}$ is denoted by $\frac{1}{v}$ then $\pi = \frac{\Delta K}{K} = \frac{s}{v}$ which is nothing but the warranted rate of growth of Harrod. Thus Robinson's growth model comes essentially to the same thing as those of Harrod and Domar. However, there is an important difference. In Robinson's model capital accumulation depends on profit wage relation (π and w/p) as well as on labour productivity (ρ). But in Harrod or Domar capital accumulation depends on the saving ratio (s) and the productivity of capital or its reciprocal (β or v). In Harrod-Domar model saving ratio is related to national income and not to profit income alone. It is also significant that while Robinson approaches the question of capital accumulation from the standpoint mainly of labour, Harrod and Domar approach it from the standpoint of capital.

Mrs. Robinson's chief contribution to post-Keynesian growth economics seems to be that she has integrated classical value and distribution theory and modern Keynesian saving-investment theory into one coherent system.

CHAPTER -19

EMBODIED TECHNICAL PROGRESS

19.1. Definition and Types of Embodied Technical Progress

Technical progress may be disembodied or embodied. In the case of disembodied technical progress it is assumed that all units of labour and capital are homogeneous and all units of labour and capital are equally affected by technical progress. Technical knowledge is assumed to fall like manna from heaven. In the case of embodied technical progress it is assumed that all units of labour and capital are not equally affected. Only certain types of capital equipment and certain sections of labour are assumed to benefit from higher productivity resulting from embodied technical progress. For example it may be assumed that technical progress is embodied in the machines of current vintage being installed and/or in the currently employed labour. Obviously we cannot make the assumption of homogeneous capital or homogeneous labour. We then have different vintages of capital or different vintages of labour. Capital goods of one vintage are different from capital goods of other vintages. Similarly workers of one vintage are also different from workers of other vintages. However, each vintage consists of a homogeneous set of machines and workers. Total capital stock is made up of machines of different vintages. Similarly total labour force is also made up of workers of different vintages.

In this approach new capital accumulation is regarded as the vehicle of technical progress. Technical progress increases the productivity of machines built in any period compared with machines built in the previous period, but it does not increase the productivity of machines already in existence. Technical progress is "embodied" in new machines. Machines built at different dates (or, machines of different vintages) are therefore qualitatively dissimilar, and cannot in the general case be aggregated into a single measure of capital. A separate production function is needed for each vintage. Total output is the sum of outputs from all the vintages in use.

In the analysis of vintage model we need two time variables, one (t) for time in the usual sense and the other (τ) for dating of vintages of machines in use at time t. In a discrete analysis, with equal spacing of vintages (say each year), the machines in use at time t are of vintages $\tau = t$ (new), $t-1$ (one year old), $t-2$ (two years old) ... etc. In continuous analysis there are machines of all vintages $\tau \leq t$ in use at any time t.

Machines may be subject to physical deterioration (depreciation) or to economic deterioration (obsolescence). In the latter, as the machines get older, their quasi-rent in the Marshallian sense falls and eventually becomes hero when the machines are scrapped. In order to simplify the analysis we assume that there is no depreciation. On the other hand it is essential to investigate obsolescence and to find the economic life (T) of machines of a particular vintage as a variable in the model. Generally machines in use at time t are of vintages τ, where $t - T \leq \tau \leq t$.

It is assumed that technical progress proceeds at a given proportional rate m but falls like manna from heaven only on new machines. Once installed, the technical features of the machines remain frozen, later technical progress passing them by. At time t, machines of vintage τ have benefited from technical progress.

We must distinguish between substitutability between machines and labour before the installation of new machines on the one hand, and that after the machines are installed on the other hand. When the machines are being designed and the decision taken to install, we may assume substitutability according to a smooth production function. In general, the function will vary from one vintage to another. At time t when the machines of vintage τ are new, the production function can be written as

$$Q_\tau = F_\tau(K_\tau, L_\tau) \quad ..... (1)$$

where K_τ is the number of machines, while L_τ and Q_τ are labour inputs and product outputs. For the sake of simplicity it is assumed that the same production function is applicable for all vintages. It is also assumed that the production function is of the Cobb-Douglas form :

$$Q_\tau = e^{\lambda\tau} K_\tau^{\alpha} L_\tau^{1-\alpha} \quad ..... (2)$$

where $\lambda > 0$, and $0 < \alpha < 1$. If the production function is of the Cobb-Douglas form :

$$Q_t = e^{\lambda t} K_t^{\alpha} L_t^{1-\alpha}, \ \lambda > 0 \text{ and } 0 < \alpha < 1,$$

it is both Harrod-neural and Solow neutral. It can be written as

$$Q_t = K_t^{\alpha} \cdot \left(e^{\frac{\lambda t}{1-\alpha}} L_t \right)^{1-\alpha} = K_t^{\alpha} (\overline{L}_t)^{1-\alpha}$$

where $\overline{L}_t = e^{\frac{\lambda}{1-\alpha}t}.L_t = e^{mt}L_t$ when $\frac{\lambda}{1-\alpha} = m$ or $\lambda = m(1-\alpha)$. Here $\overline{L}_t$ is labour input in efficiency units. Now $m = \frac{\lambda}{1-\alpha}$ represents the rate of Harrod-neutral technical progress. Similarly the production function can be written as

$$Q_t = \left(e^{\frac{\lambda}{\alpha}t} K_t\right)^{\alpha}.L_t^{1-\alpha}$$

$= \left(\overline{K}_t\right)^{\alpha} L_t^{1-\alpha}$ where $\overline{K}_t = e^{\frac{\lambda}{\alpha}t} K_t$ represents machine in efficiency units. If m' is the rate of Solow-neutral technical progress, then $m' = \frac{\lambda}{\alpha} = \frac{m(1-\alpha)}{\alpha}$. Thus though the Cobb-Douglas production function represents both Harrod-neutral and Solow-neutral technical progress, the rate of technical progress are not the same. While $m = \frac{\lambda}{1-\alpha}$ is the rate of Harrod-neutral technical progress $m' = \frac{\lambda}{\alpha} = \frac{m(1-\alpha)}{\alpha}$ is the rate of Solow-neutral technical progress.

It is a different question how far machines and labour can be substituted at any time $t > \tau$ after the machines of vintage τ are installed. At time $t=\tau$, a decision is taken to use a labour crew of L_τ with K_τ machines of new vintage, the output Q_τ being given by (2). At any time $t>\tau$, two alternative cases are possible : *either* substitution between capital and labour continues according to (2) *or* the K_τ machines installed at time τ continue to be used with the same labour crew L_τ that goes with them. On the first alternative the employment of machines of τ and the labour used with them can and does vary over time. On the second alternative, once the machines are installed, they remain for all time (until scrapped as obsolescent) with the same team of men and producing the same output.

Following Phelps we can distinguish the two alternative cases as *putty-putty* and *putty-clay*. The *putty-putty case* has smooth substitution both before and after installation and according to the same production function. The *putty-clay case* allows for substitution of labour for new machines ; but once installed a machine is operated by a fixed labour crew. There is a special case of the latter which may be called the *clay-clay case* where capital-labour ratio is fixed both before and after installation. Instead of smooth production function we then have a fixed coefficient production function.

19.2. Vintage Capital with Factor Substitution

Consider the putty-putty case where there is substitution between labour and capital at all times, before and after installation. Machines are installed in a continuous sequence of vintages and the number of machines of vintage τ is K_τ varying with τ. Here K_τ is the rate of installation per unit of time so that the number of machines brought in during the time-interval τ to $\tau + d\tau$ is $K_\tau d\tau$. It is assumed that the machines are of infinite life and the number of machines of vintage τ in use at time t remains as K_τ for all $t \geq \tau$. The production function is assumed to be of Cobb-Douglas form for all vintages. It is given by

$$Q_\tau(t) = e^{m(1-\alpha)\tau} K_\tau^\alpha \{L_\tau(t)\}^{1-\alpha} \text{ for } t \geq \tau \text{. ..... (3)}$$

It implies that technical progress at the Harrod neutral rate m is operative upto the time τ at which the machines are brought in, but not thereafter. The number of machines of vintage τ is use remains constant at K_τ. The time variables in the above production function are, therefore, $L_\tau(t)$ and $Q_\tau(t)$.

Labcur is allocated to machines under perfect competition, the wage rate $w(t)$ varies over time, but applies to labour used with machines of all vintages. Hence $w(t)$ equals the marginal product of labour for machines of each vintage at time t.

$$i.e. \quad \frac{\partial Q_\tau(t)}{\partial L_\tau(t)} = \omega(t) \text{ for each } \tau \text{ and } t \geq \tau.$$

From (3) we get $\dfrac{\partial Q_\tau(t)}{\partial L_\tau(t)} = (1-\alpha)\dfrac{Q_\tau(t)}{L_\tau(t)}$.

$$\text{Hence } (1-\alpha)\frac{Q_\tau(t)}{L_\tau(t)} = \omega(t) \text{ for each } \tau \text{ and } t \geq \tau \text{ ..... (4)}$$

The allocation of labour follows from (3) and (4) which give both $Q_\tau(t)$ and $L_\tau(t)$ in terms of the ruling wage rate $w(t)$.

From (3)

$$Q_\tau(t) = e^{m(1-\alpha)} K_\tau^\alpha \{L_\tau(t)\}^{1-\alpha}$$

But from (4) $L_\tau(t) = \dfrac{(1-\alpha)Q_\tau(t)}{\omega(t)}$

From (3) and (4) we get

$$Q_\tau(t) = e^{m(1-\alpha)\tau} K_\tau^\alpha \left\{\frac{(1-\alpha)Q_\tau(t)}{\omega(t)}\right\}^{1-\alpha}$$

$$\text{or, } \{Q_\tau(t)\}^\alpha = e^{m(1-\alpha)\tau} K_\tau^\alpha (1-\alpha)^{(1-\alpha)} \{w(t)\}^{-(1-\alpha)}$$

$$\therefore\ Q_\tau(t) = e^{\frac{m(1-\alpha)}{\alpha}\tau} . K_\tau . (1-\alpha)^{\frac{1-\alpha}{\alpha}} . \{\omega(t)\}^{-\left(\frac{1-\alpha}{\alpha}\right)}$$

$$\text{or, } Q_\tau(t) = e^{m'\tau} . (1-\alpha)^{\frac{1-\alpha}{\alpha}} . \{\omega(t)\}^{\frac{-(1-\alpha)}{\alpha}} . K_\tau \quad \ldots\ldots (5)$$

where $m' = \dfrac{m(1-\alpha)}{\alpha}$.

$$\text{Again } L_\tau(t) = \frac{(1-\alpha)Q_\tau(t)}{\omega(t)}$$

$$= \frac{(1-a)}{\omega(t)} e^{m(1-\alpha)\tau} K_\tau^\alpha \{L_\tau(t)\}^{1-\alpha}$$

$$\text{or, } \{L_\tau(t)\}^\alpha = \frac{(1-\alpha)}{\omega(t)} e^{m(1-\alpha)\tau} K_\tau^\alpha$$

$$\therefore\ L_\tau(t) = (1-a)^{\frac{1}{\alpha}} . \{\omega(t)\}^{-\frac{1}{\alpha}} . e^{\frac{m(1-\alpha)}{\alpha}\tau} . K_\tau$$

$$\text{or, } L_\tau(t) = e^{m'\tau} . (1-\alpha)^{\frac{1}{\alpha}} . \{\omega(t)\}^{-\frac{1}{\alpha}} . K_\tau \quad \ldots\ldots (6)$$

From (5) and (6) it is seen that labour and output per machine for any vintage τ depend only on the varying wage rate overtime. If $w(t)$ increases overtime then the allocation of labour to a machine of a given vintage declines and, with it, output from the machine also declines.

If $Q(t)$ is the output obtained from all machines and $L(t)$ is the labour employed, at time t, then by integration

$$Q(t) = (1-\alpha)^{\frac{1-\alpha}{\alpha}} \{\omega(t)\}^{\frac{-(1-\alpha)}{\alpha}} \int_{-\infty}^{t} e^{m'\tau} K_\tau . d\tau$$

$$\text{and } L(t) = (1-\alpha)^{\frac{1}{\alpha}} . \{\omega(t)\}^{-\frac{1}{\alpha}} . \int_{-\infty}^{t} e^{m'\tau} K_\tau . d\tau$$

$$\therefore \frac{Q(t)}{L(t)} = \frac{1}{1-\alpha}\omega(t).$$

Here $Q(t)$, $L(t)$ and $w(t)$ are time-variables. We can drop the explicit reference to t and write simply Q, L and w.

Then $Q = (1-\alpha)^{\frac{1-\alpha}{\alpha}} . \{\omega\}^{-\frac{-(1-\alpha)}{\alpha}} . J$ ..... (7)

$L = (1-\alpha)^{\frac{1}{\alpha}} . \{\omega\}^{-\frac{1}{\alpha}} . J$ ..... (8) where

$J = \int_{-\infty}^{t} e^{m'\tau} . K_\tau . d\tau$ is obtained from the numbers of machines of various vintages. Further :

$\omega = (1-\alpha)\dfrac{Q}{L}$ ..... (9)

for the wage rate. It should be noted that Q, L and w as given by (7), (8) and (9) are variables over time.

19.3. The Aggregate Production Function

To derive the aggregate production function we have to obtain measures to aggregate capital stock, aggregate output and aggregate labour input. Aggregate capital stock at historical cost is given by

$$N(t) = \int_{-\infty}^{t} K_\tau . d\tau$$

This total is, however, not an adequate representation of capital stock because machines of successive vintages become more efficient with technical progress. Later machines must count for more than earlier ones. We, therefore, take a weighted sum of the numbers of machines of different vintages and the weight of K_τ is $e^{m'\tau}$ where m' is the rate of Solow-neutral technical progress. Hence, the aggregate capital stock in efficiency units at time t is

$\overline{K}(t) = \int_{-\infty}^{t} \overline{K}_\tau . d\tau = \int_{-\infty}^{t} e^{m'\tau} . K_\tau . d\tau$ ..... (10)

which is nothing but J in (7) and (8). Then from (7) and (8) we get

$Q = (1-\alpha)^{\frac{1-\alpha}{\alpha}} . \{\omega\}^{\frac{-(1-\alpha)}{\alpha}} . \overline{K}$ and

$L = (1-\alpha)^{\frac{1}{\alpha}} . \{\omega\}^{\frac{-1}{\alpha}} . \overline{K}$

Eliminating w we get

$$\frac{Q}{L^{1-\alpha}} = \frac{(1-\alpha)^{\frac{1-\alpha}{\alpha}} w^{\frac{-(1-\alpha)}{\alpha}} \overline{K}}{(1-\alpha)^{\frac{1-\alpha}{\alpha}} w^{\frac{-(1-\alpha)}{\alpha}} \overline{K}^{1-\alpha}} = \frac{\overline{K}}{(\overline{K})^{1-\alpha}} = (\overline{K})^{\alpha}$$

$$\therefore\ Q = (\overline{K})^{\alpha} . L^{1-\alpha} \quad ..... (11)$$

This is the aggregate production function. It is of Cobb-Douglas form. Here technical progress is embodied in $\overline{K}$ which represents capital measured in efficiency units.

Alternatively if capital is measured in fixed units the aggregate production function can be written as

$$Q = e^{m'\alpha t} K^{\alpha} L^{1-\alpha} \quad ..... (12) \text{ where } m' = \frac{m(1-\alpha)}{\alpha}$$

This is also Cobb-Douglas production function with Solow-neutral technical progress at the rate m'.

Under perfect competition the marginal products of the production function (12) equal the profit rate (ρ) and the wage rate (w) at time t :

$$\rho = \frac{\partial Q}{\partial K} = \alpha \frac{Q}{K} \text{ and}$$

$$w = \frac{\partial Q}{\partial L} = (1-\alpha)\frac{Q}{L}.$$

Further $Q = \rho K + wL$ at time t. These results are the same as for any Cobb-Douglas production function.

19.4. Vintage Model of Growth with Factor Substitution (Putty-Putty Model)

Consider the conditions of equilibrium in the product market and the labour market in a vintage model. For full capacity total output is equal to total income. If Q represents total output and Y represents total income then full capacity requires :

$$Q = Y = \overline{K}^{\alpha} . L^{1-\alpha} \quad ..... (1)$$

where $\overline{K} = e^{m't} . K$

Flow condition of equilibrium in the product market requires $S=I$.

The saving function is assumed to be proportional $S = sY$ where $0 < s < 1$. Investment at time t is the number of new machines of vintage $\tau = t$ installed : $I = K_t$. Therefore equilibrium requires $I = sY$. Aggregate capital stock in efficiency units at time t is given by

$$\overline{K}(t) = \int_{-\infty}^{t} \overline{K}_\tau . d\tau = \int_{-\infty}^{t} e^{m'\tau} . K_\tau . d_\tau$$

$$\therefore \frac{d\overline{K}(t)}{dt} = e^{m't} K_t = e^{m't} I$$

$$\therefore I = e^{-m't} \frac{d\overline{K}}{dt}.$$

Flow condition of equilibrium in the product market requires

$$e^{-m't} \frac{d\overline{K}}{dt} = sY \quad ..... (2)$$

Full-employment in the labour market requires $L = L_0 e^{nt}$ ..... (3) where supply of labour is assumed to be growing at a given rate, n.

Hence, the three conditions of equilibrium in this vintage model of putty-putty type are :

$$Y = \overline{K}^\alpha . L^{1-\alpha} \quad ..... (1)$$

$$I = e^{-m't} \frac{d\overline{K}}{dt} = sY \quad ..... (2)$$

$$L = L_0 e^{nt} \quad ..... (3)$$

These 3 equations can be condensed into a single equation :

$$\frac{d\overline{K}}{dt} = s e^{m't} \overline{K}^\alpha . L_0^{1-\alpha} . e^{n(1-\alpha)t}$$

$$\text{or, } \frac{d\overline{K}}{dt} = s L_0^{1-\alpha} . e^{\{m'+n(1-\alpha)\}t} \overline{K}^\alpha \quad ..... (4)$$

This is a difference equaticn in $\overline{K}$. Solving this equation we can get the time path of $\overline{K}$. Equation (4) can be solved as follows : From (4) we get

$$\left(\overline{K}\right)^{-\alpha} . d\overline{K} = s L_0^{1-\alpha} . e^{\{m'+n(1-\alpha)\}t} . dt$$

Integrating both sides

$$\int \left(\overline{K}\right)^{-\alpha} . d\overline{K} = s L_0^{1-\alpha} \int e^{\{m'+n(1-\alpha)\}t} . dt$$

$$\text{or, } \frac{1}{1-a}\left(\overline{K}\right)^{1-\alpha} = s L_0^{1-\alpha} \frac{1}{m'+n(1-\alpha)} . e^{\{m'+n(1-\alpha)\}t}$$

$$\therefore \left(\overline{K}\right)^{1-\alpha} = \frac{s L_0^{1-\alpha}(1-\alpha)}{m'+n(1-\alpha)} . e^{\{m'+n(1-\alpha)\}t}$$

$$\therefore\ \bar{K} = L_0 \left\{ \frac{s}{\frac{m'}{1-\alpha} + n} \right\}^{\frac{1}{1-\alpha}} . e^{\left\{\frac{m'}{1-\alpha}+n\right\}t}$$

or, $\bar{K} = \bar{K}_0 . e^{\left\{\frac{m'}{1-\alpha}+n\right\}t}$ where

$$\bar{K}_0 = L_0 \left\{ \frac{s}{\frac{m'}{1-\alpha} + n} \right\}^{\frac{1}{1-\alpha}} \quad ..... (5)$$

This gives the time path of capital stock in efficiency units. The corresponding time path of capital in fixed units is given by

$K = e^{-m't}\bar{K}$ and $K_0 = \bar{K}_0$

or, $K = K_0 e^{\left\{\frac{m'}{1-\alpha}+n-m'\right\}t} = K_0 . e^{\left\{\frac{m'\alpha}{1-\alpha}+n\right\}t}$ ..... (6)

The time path of output comes from the production function

$$Y = \bar{K}^{\alpha} L^{1-\alpha}$$

$$= \bar{K}_0^{\alpha} . e^{\left\{\frac{m'\alpha}{1-\alpha}+n\right\}\alpha t} . L_0^{1-\alpha} . e^{n(1-\alpha)t}$$

$$= \left(\bar{K}_0^{\alpha} . L_0^{1-\alpha}\right) . e^{\left\{\frac{m'\alpha}{1-\alpha}+n\right\}t}$$

$$= Y_0 . e^{\left\{\frac{m'\alpha}{1-\alpha}+n\right\}t} \quad ..... (17)$$

where $Y_0 = L_0 \left\{ \frac{s}{\frac{m'}{1-\alpha} + n} \right\}^{\frac{\alpha}{1-\alpha}}$

Finally, the steady state path of investment I which is the number of new machines installed at time t is obtained from $I = sY$.

The results have been expressed in terms of Solow-neutral rate of technical progress m'. They can be expressed in terms of Harrod-neutral rate of technical progress m with the help of the relation

$$m = \frac{m'\alpha}{1-\alpha}.$$

Then $K = K_0 e^{(m+n)t}$ ..... (8)

and $Y = Y_0 e^{(m+n)t}$ ..... (9)

This shows that the putty-putty vintage model produces a result, at the end, which is in line with the neo-classical model with disembodied technical progress. There is a steady state solution, both K and Y growing at the rate $\mu = m+n$ provided the initial values are right. The initial output capital ratio must be

$$\frac{Y_0}{K_0} = \frac{m+m'+n}{s} \quad ..... (10)$$

and it then remains constant in the steady state growth.

The steady state rate of growth of the model is $\mu = m+n$, involving the Harrod-neutral rate of technical progress m. But both m and the Solow-neutral rate m' appear in the initial conditions :

$$K_0 = L_0\left(\frac{s}{m+m'+n}\right)^{\frac{1}{1-\alpha}} \text{ and}$$

$$Y_0 = L_0\left(\frac{s}{m+m'+n}\right)^{\frac{\alpha}{1-\alpha}}$$ and in the output-capital ratio. The influence of the propensity to save (s) is only on the levels of output and capital and not on their rates of growth.

It is essential to appreciate that the result depends on the assumption of a Cobb-Douglas production function. This assumption permits technical progress both at the Harrod-neutral rate m and at the Solow-neutral rate $m' = \dfrac{m(1-\alpha)}{\alpha}$. Indeed, the Cobb-Douglas form is the only production function with this property essential to the vintage model. Harrod neutral technical progress is needed to make steady state growth at the rate $(m+n)$, possible. Solow-neutral technical progress is needed in the construction of a measure of capital stock K and to make an aggregate production function possible.

19.5. The Capital Stock and its Valuation

At any time t on the steady state growth path, the number of machines to vintage t in the capital stock is

$K_\tau = sY(\tau) = sY_0 . e^{\mu\tau}$ for $\tau \le t$ where $\mu = m+n$.

The total number of machines is

$$N(t) = \int_{-\infty}^{t} K_\tau d\tau = sY_0 \int_{-\infty}^{t} e^{\mu\tau} d\tau$$

$$= \frac{sY_0}{\mu} e^{\mu t} \quad ..... (1)$$